Cambridge Assessment
Admissions Testing

NAZARBAYEV
UNIVERSITY

Foundation Year Program

Entrance Tests

MATHEMATICS

SAMPLE QUESTIONS WITH SOLUTIONS

For

NUFYP SET 2021

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Question 1

Which one of the following is a simplification of

$$y^2\sqrt{x}\left(\frac{2x^{\frac{3}{2}}}{y^{\frac{1}{2}}}\right)^3$$

A $2x^5y^{\frac{1}{2}}$

B $8x^5y^{\frac{3}{2}}$

C $8x^{\frac{9}{2}}y^{\frac{1}{2}}$

D $2x^5y^{\frac{3}{2}}$

E $8x^5y^{\frac{1}{2}}$

Explanation 1

If we expand the contents of the brackets and rewrite $\sqrt{x}$ as an exponent, we get:

$$y^2\sqrt{x}\left(\frac{2x^{\frac{3}{2}}}{y^{\frac{1}{2}}}\right)^3 = y^2x^{\frac{1}{2}} \times \frac{\left(2x^{\frac{3}{2}}\right)^3}{\left(y^{\frac{1}{2}}\right)^3}$$

By the laws of indices:

$$= y^2x^{\frac{1}{2}} \times \frac{2^3x^{\frac{3}{2}\times 3}}{y^{\frac{1}{2}\times 3}} = y^2x^{\frac{1}{2}} \times \frac{8x^{\frac{9}{2}}}{y^{\frac{3}{2}}} = \frac{8x^{\frac{1}{2}+\frac{9}{2}}y^2}{y^{\frac{3}{2}}} = 8x^5y^2 \times y^{-\frac{3}{2}} = 8x^5y^{2-\frac{3}{2}} = 8x^5y^{\frac{1}{2}}$$

So the answer is E.

Question 2

$PQRS$ is a square.

Sides PQ and RS are each increased in length by 40%.

Sides QR and PS are each decreased in length by 75%.

$PQRS$ has been transformed into a rectangle.

What is the percentage decrease in the area caused by the transformation?

A 10%

B 30%

C 35%

D 65%

E 70%

F 90%

Explanation 2

Call the sides of $PQRS$ x, so its area is $x \times x = x^2$

If PQ and RS increase in length by 40%, their new lengths are 140% of their original length, and so could be written as $1.4x$

If QR and PS decrease in length by 75%, their new lengths are 25% of their original length, and so could be written as $0.25x$

Therefore, the new area can be written as:

$$0.25x \times 1.4x = \frac{1}{4}x \times \frac{7}{5}x = \frac{7}{20}x^2$$

The new area is $\frac{7}{20} \times 100 = \frac{70}{2} = 35\%$ of the original area.

This is a 65% decrease in size, and so the answer is D.

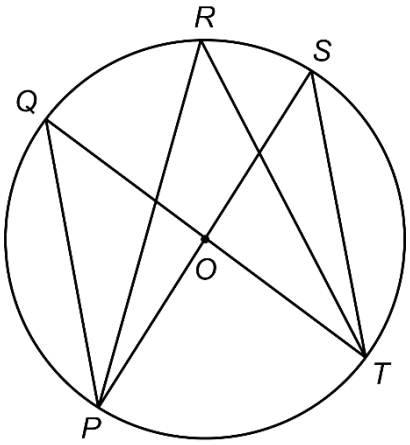
Question 3

P, Q, R, S and T are points on the circumference of a circle.

O is the centre of the circle.

Angle $POQ = (4x - 10)°$

Angle $SOT = (3x + 20)°$



[diagram not to scale]

What is the size of angle PRT ?

A 25°

B 30°

C 35°

D 55°

E 70°

F 75°

G 110°

H 140°

Explanation 3

Vertically opposite angles around a point are equal, so:

$$4x - 10 = 3x + 20$$
$$x = 30$$

Therefore angle $POQ = 4 \times 30 - 10 = 120 - 10 = 110$

Angles on a straight line sum $180°$, so $QOS = 180 - 110 = 70$

A circle theorem says that angles subtended at the centre (in this case, POT) are twice the angle subtended at the circumference (in this case, PRT). That is,

$$PRT = \frac{70}{2} = 35$$

And the answer is C.

Question 4

Consider the simultaneous equations:

$$3x = y + 3$$

$$4^{x+1} = \frac{16^{y+1}}{8^{y+3}}$$

What is the value of $x - y$?

A -17

B -11

C -5

D -3

E 3

F 5

G 11

H 17

Explanation 4

We can rearrange $3x = y + 3$ to make y the subject: $y = 3x - 3$

Call this equation (1).

In the second equation, write everything as a power of 2:

$$4^{x+1} = \frac{16^{y+1}}{8^{y+3}}$$
$$(2^2)^{x+1} = \frac{(2^4)^{y+1}}{(2^3)^{y+3}}$$

Expanding these brackets:

$$2^{2x+2} = \frac{2^{4y+4}}{2^{3y+9}} = 2^{(4y+4)-(3y+9)} = 2^{y-5}$$

Therefore,

$$2x + 2 = y - 5$$
$$y = 2x + 7$$

Call this equation (2).

So we now have two simpler equations:

$$y = 3x - 3 \text{ (1)}$$
$$y = 2x + 7 \text{ (2)}$$

Therefore, substituting (1) into (2) we get:

$$3x - 3 = 2x + 7$$
$$x = 10$$

Since $y = 3x - 3 = (3 \times 10) - 3 = 30 - 3 = 27$

Therefore $x - y = 10 - 27 = -17$ and the answer is A.

Question 5

A chemist needs to mix together two chemicals, P and Q, in the volume ratio 5 **:** 2

The chemist has two mixtures, X and Y:

- Mixture X contains P and Q in the volume ratio 3 **:** 2
- Mixture Y contains P and Q in the volume ratio 4 **:** 1

The chemist combines mixtures X and Y in order to achieve the required volume ratio of P and Q.

What is the resulting volume ratio of X to Y?

A 3 **:** 4

B 4 **:** 7

C 5 **:** 9

D 13 **:** 16

E 14 **:** 17

Explanation 5

We are told:

- Mixture X contains P and Q in the volume ratio 3 **:** 2
- Mixture Y contains P and Q in the volume ratio 4 **:** 1

We will mix X and Y in the ratio a **:** b

We need to work out what a **:** b is to make the final ratio of P to Q equal to 5 **:** 2

Now, (a lots of X) + (b lots of Y) gives us a ratio of P to Q of $(3a + 4b)$ **:** $(2a + b)$

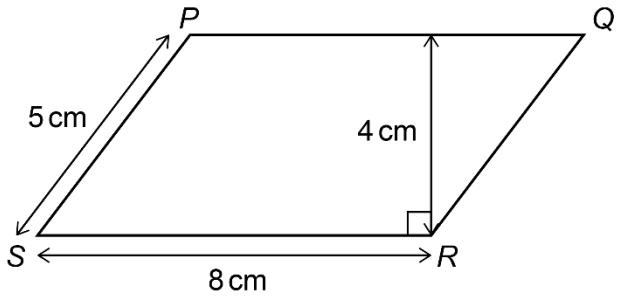
So, we need to find what ratio a **:** b makes $(3a + 4b)$ **:** $(2a + b)$ the same as the ratio 5 **:** 2

This means we need a **:** b such that $\frac{3a+4b}{5} = \frac{2a+b}{2}$

Rearranging this gives: $\frac{a}{b} = \frac{3}{4}$

Therefore the ratio is 3 **:** 4 so the answer is A.

Question 6



[diagram not to scale]

This parallelogram PQRS undergoes an enlargement with scale factor $\frac{1}{4}$

What are the perimeter and area of the parallelogram after the enlargement?

	Perimeter after enlargement	**Area after enlargement**
A	6.5 cm	2 cm^2
B	6.5 cm	2.5 cm^2
C	6.5 cm	2.75 cm^2
D	6.5 cm	8 cm^2
E	8 cm	2 cm^2
F	8 cm	2.5 cm^2
G	8 cm	2.75 cm^2
H	8 cm	8 cm^2

Explanation 6

Initially,

$$\text{Perimeter of } PQRS = \text{sum of lengths of each side} = 8 + 5 + 8 + 5 = 26 \text{ cm}$$

$$\text{Area of } PQRS = \text{base} \times \text{height} = 8 \times 4 = 32 \text{ cm}^2$$

An enlargement of scale factor $\frac{1}{4}$ means that all lengths are divided by 4.

Therefore $RS = PQ$ becomes 2, $PS = QR$ becomes $\frac{5}{4}$ and the height becomes 1.

Now,

$$\text{Perimeter of } PQRS = \text{sum of lengths of each side} = 2 + \frac{5}{4} + 2 + \frac{5}{4} = 4 + \frac{5}{2} = 4 + 2.5 = 6.5 \text{ cm}$$

$$\text{Area of } PQRS = \text{base} \times \text{height} = 2 \times 1 = 2 \text{ cm}^2$$

Therefore the perimeter is 6.5 cm and the area is 2 cm^2 so the answer is A.

Question 7

P is inversely proportional to the square root of Q.

$P = 2$ when $Q = 9$

R is inversely proportional to Q.

$R = 4$ when $Q = 3$

Which one of the following is a correct equation for R in terms of P?

A $R = \frac{P}{3}$

B $R = \frac{2P}{3}$

C $R = \frac{P^2}{3}$

D $R = 2P^2$

E $R = \frac{16}{3P^2}$

F $R = \frac{48}{P^2}$

G $R = 3P^2$

Explanation 7

If P is inversely proportional to the square root of Q, then we can write this as $P = \frac{k_1}{\sqrt{Q}}$ for some unknown constant k_1

When $P = 2$, $Q = 9$ so we can use this to find out what k_1 is:

$$2 = \frac{k_1}{\sqrt{9}}$$
$$2 = \frac{k_1}{3}$$
$$k_1 = 2 \times 3 = 6$$

Therefore $P = \frac{6}{\sqrt{Q}}$

If R is inversely proportional to Q, then we can write this as $R = \frac{k_2}{Q}$ for some other constant k_2

When $R = 4$, $Q = 3$ so we can find out what k_2 is:

$$4 = \frac{k_2}{3}$$
$$k_2 = 4 \times 3 = 12$$

Therefore $R = \frac{12}{Q}$

We want R in terms of P, so we can rearrange the formula for P to get:

$$P = \frac{6}{\sqrt{Q}}$$
$$\sqrt{Q} = \frac{6}{P}$$
$$Q = \frac{6^2}{P^2} = \frac{36}{P^2}$$

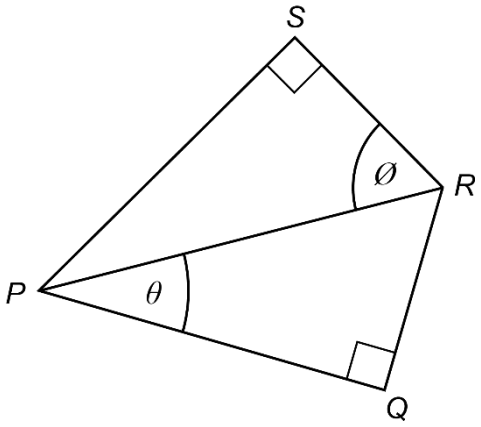
Then, using the formula for R we get:

$$R = 12 \div \frac{36}{P^2} = \frac{12P^2}{36} = \frac{P^2}{3}$$

And the answer is C.

Question 8

Two right-angled triangles are joined together to make a quadrilateral *PQRS,* as shown in the diagram.



[diagram not to scale]

$\tan\theta = \frac{3}{4}$

$\sin\emptyset = \frac{7}{9}$

Find the ratio of length PQ **:** length PS

A 4 **:** 7

B 28 **:** 45

C 20 **:** 63

D 27 **:** 28

E 27 **:** 35

F 12 **:** 7

G 36 **:** 35

Explanation 8

Using trigonometric identities, we can work out that:

$$\tan\theta = \frac{3}{4} = \frac{QR}{PQ}$$

And

$$sin\phi = \frac{7}{9} = \frac{PS}{PR}$$

That is, the lengths of QR and PQ are some multiple x of 3 and 4, respectively.

Similarly, the lengths of PS and PR are some multiples y of 7 and 9, respectively.

Using Pythagoras, we can calculate PR:

$$(PR)^2 = (4x)^2 + (3x)^2 = 16x^2 + 9x^2 = 25x^2$$

$$PR = \sqrt{25x^2} = 5x$$

We now have $PR = 5x = 9y$ so $y = \frac{5}{9}x$

We want the ratio of the length of PQ: length of PS so:

$$4x : 7y$$

$$4x : 7\left(\frac{5}{9}x\right)$$

$$4x : \frac{35}{9}x$$

$$36x : 35x$$

$$36 : 35$$

And the answer is G.

The Nazarbayev University Foundation Year Program entrance test was developed in collaboration with Cambridge Assessment Admissions Testing, part of the University of Cambridge (2020-2021).